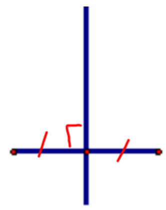
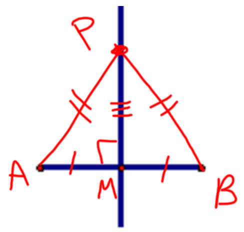
Ch 5 Relationships in Triangles
5.1 Bisectors, Medians, and Altitudes

perpendicular bisector-of a side of a triangle is a line segment or ray that passes through the midpoint of the side and is perpendicular to it.



Thm 5.1-Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

PA = PB

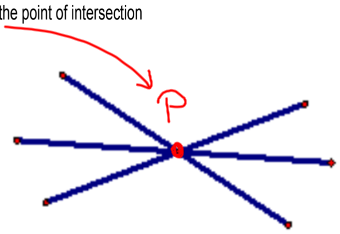


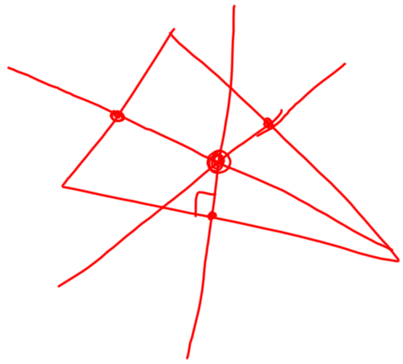
Thm 5.2-Any point equidistant from the endpoints of the segment lies on the perpendicular bisector of the segment.

Converse

Concurrent lines-three or more lines that intersect at a common point

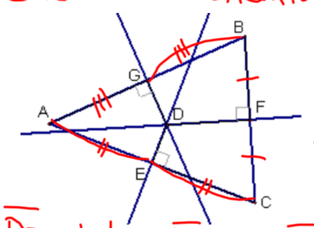
Point of concurrency-the point of intersection

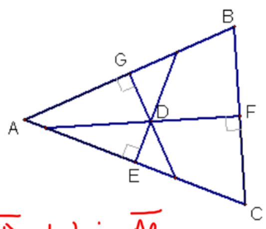




Circumcenter-the point of concurrency of the perpendicular bisectors of a triangle.

D is the circumcenter



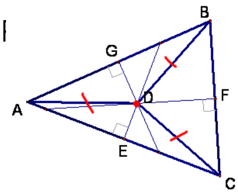


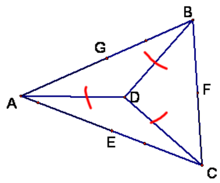
$\overline{DE}$ ⊥ bis $\overline{BC}$

DE ⊥ bis $\overline{AC}$

$\overline{GD}$ ⊥ bis $\overline{AB}$

Thm 5.3 Circumcenter theorem-The circumcenter of a triangle is equidistant from the vertices of the triangle.



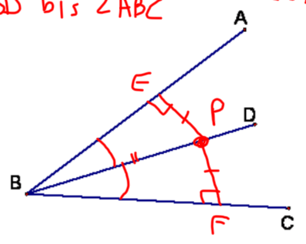


DA = DC = DB

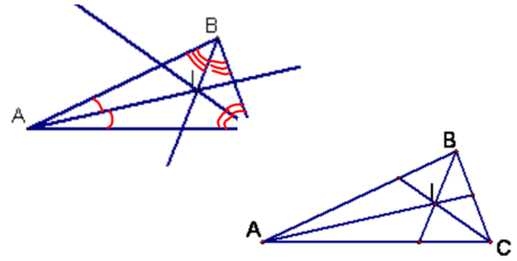
Thm 5.4-Any point on the angle bisector is equidistant from the sides of the angle.

Thm 5.5-Any point equidistant from the sides of an angle lies on the angle bisector.

$\overline{BD}$ bis $\angle ABC$ Concl: PE = PF

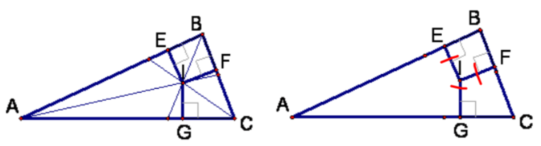


Incenter-The point of concurrency of the angle bisectors of a triangle.



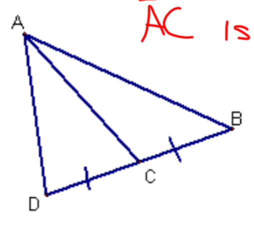
Thm 5.6 The Incenter Theorem-The incenter of a triangle is equidistant from each side of the triangle.

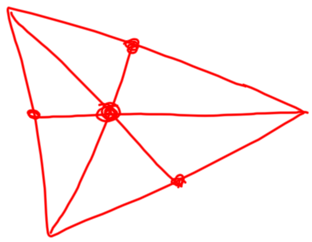
Concl: IE = IF = IG



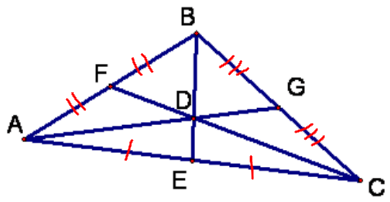
Median-is a segment whose endpoint are the vertex of a triangle and the midpoint of the side opposite the vertex.

$\overline{AC}$ is a median





Centroid-The point of concurrency of the three medians of a triangle.



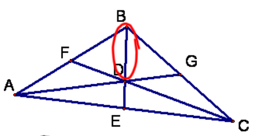
Thm 5.7 The Centroid Theorem-The centroid of a triangle is located two-thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.

Conclusion:

DB = 2/3 BE DE = 1/3 BE

DA = 2/3 AG DG = 1/3 AG

CD = 2/3 CF FD = 1/3 CF

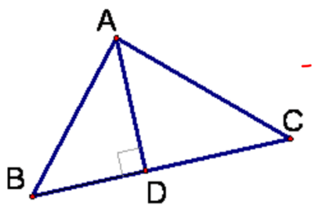


DE = $\frac{1}{2}$ BD

DG = $\frac{1}{2}$ DA

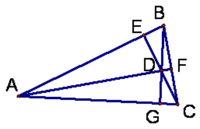
DF = $\frac{1}{2}$ DC

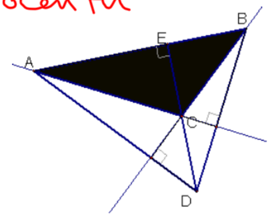
Altitude-of a triangle is a segment form a vertex to the line containing the opposite side and perpendicular to the line containing that side.



Orthocenter-The point of concurrency of the three altitudes of a triangle.

D is the orthocenter

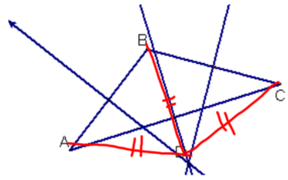


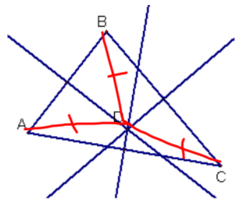


Summary

The circumcenter. (3 ⊥ bis.)

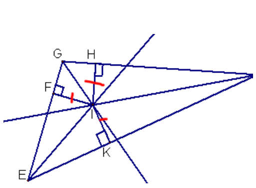
$\overline{DA} \cong \overline{DB} \cong \overline{DC}$

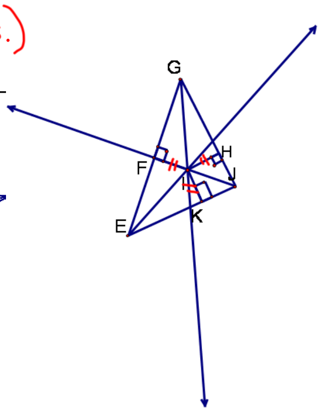




The incenter. (3 ∠ Bis.)

$\overline{IK} \cong \overline{IF} \cong \overline{IH}$



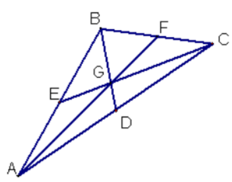


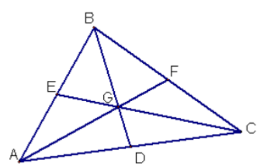
The Centroid (3 medians)

$BG = \frac{2}{3}BD$ $CG = \frac{2}{3}CE$ $AG = \frac{2}{3}AF$

$GD = \frac{1}{3}BD$ $GE = \frac{1}{3}CE$ $GF = \frac{1}{3}AF$

$GD = \frac{1}{2}BG$ $GE = \frac{1}{2}CG$ $GF = \frac{1}{2}AG$



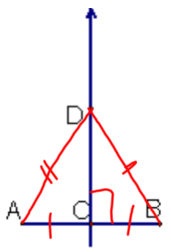


Orthocenter (3 altitudes)

Examples:

1. $\overline{CD}$ is a $\perp$ bisector of $\overline{AB}$.
$m\angle DCA = 2x$. Solve for x.____ 45

AC = 3y + 2, BC = 14. Solve for y._____ 4

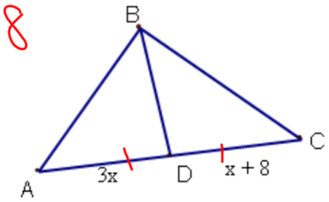


$2x = 90$
$x = 45$

$3y + 2 = 14$
$3y = 12$
$y = 4$

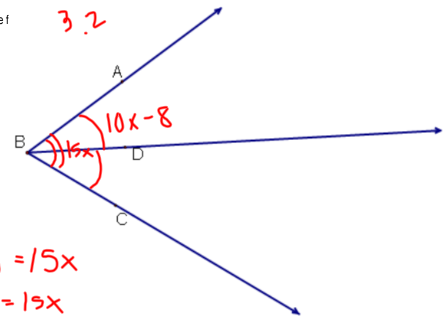
2. $\overline{BD}$ is a median in $\triangle ABC$. Solve for x._______ 4

$3x = x + 8$
$2x = 8$
$x = 4$



3. $\overline{BD}$ bisects $\angle ABC$. Solve f 3.2

$m\angle ABC = 15x$
$m\angle ABD = 10x - 8$

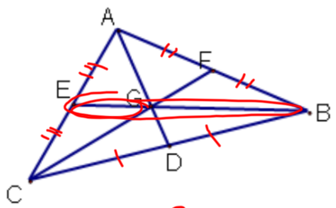


$2(10x - 8) = 15x$
$20x - 16 = 15x$
$-16 = -5x$
$3.2 = x$

4. G is the centroid.

AG = 7.4
AD = 6a
a = 1.85

GE = 5c
EB = 22.8
c = 1.52



$AG = \frac{2}{3} AD$

$7.4 = \frac{2}{3} 6a$

$7.4 = 4a$

$1.85 = a$

$GE = \frac{1}{3} EB$

$5c = \frac{1}{3} 22.8$

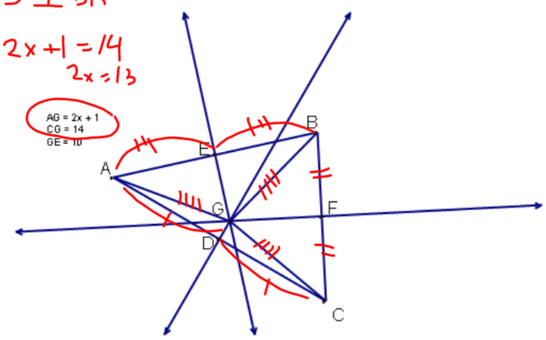
$c = 1.52$

5. G is the circumcenter. x = 6.5

3 ⊥ bis

$2x + 1 = 14$

$2x = 13$

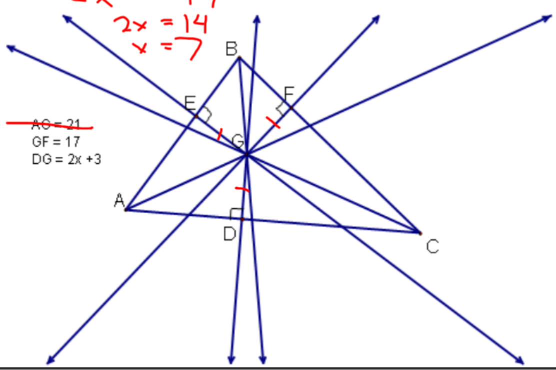


6. G is the incenter. x = 7

$2x + 3 = 17$

$2x = 14$

$x = 7$



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